

Supermassive dark-matter Q-balls in galactic centers?

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Though widely accepted, it is not proven that supermassive compact objects (SMCOs) residing in galactic centers are black holes. In particular, the Milky Way's SMCO can be a giant nontopological soliton, Q-ball, made of a scalar field: this fits perfectly all observational data. Similar but tiny Q-balls produced in the early Universe may constitute, partly or fully, the dark matter. This picture explains in a natural way, why our SMCO has very low accretion rate and why the observed angular size of the corresponding radio source is much smaller than expected. Interactions between dark-matter Q-balls may explain how SMCOs were seeded in galaxies and resolve well-known problems of standard (non-interacting) dark matter.

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Introduction and summary. — Firm observational results indicate the existence of supermassive compact objects (SMCOs) in galactic centers, and it is often assumed that these objects are black holes. However, there exist observations whose explanation requires invoking very complicated models within the black-hole paradigm. Discussed in more detail below, they include extremely inefficient accretion on the Milky-Way SMCO, very small size of the resolved Galactic-center radio source, presence of SMCOs at early stages of galactic evolution etc. Here I show that giant nontopological solitons, Q-balls, made of a scalar field not interacting with baryons represent a viable model for SMCOs, alternative to black holes. This model fits well all observational data related to the innermost part of the Milky-Way nucleus and explains its low accretion efficiency and the small angular size of the Sgr A* radio source. This is because the SMCO Q-balls may be considerably larger than a black hole of the same mass, while baryonic matter penetrates the Q-ball freely and normally passes through, and only a small fraction of baryons loses the angular momentum in collisions with each other and gets gravitationally trapped in the central part of the SMCO. At some stage, this mass gain may force the Q-ball to collapse into a black hole, giving rise to a temporal burst of the galaxy's activity. While a detailed quantitative analysis requires complicated large-scale numerical simulations, far beyond the scope of this Letter, order-of-magnitude estimates suggest that smaller Q-balls made of the same scalar field ϕ may be produced in the early Universe in the amount and with cross sections relevant for self-interacting dark matter. They may give birth to supermassive Q-balls in galactic centers via the gravothermal collapse, helping at the same time to alleviate some problems of the standard, non-interacting dark matter. The rest of dark matter may be constituted either of the ϕ particles, or of similar Q-balls of different size and, hence,

of different cross section.

Supermassive central objects. — Thanks to intense development of observational techniques, enormous amount of information about galactic central objects have been obtained in recent years. The best-studied central object resides in our own Galaxy, the Milky Way. Its observations revealed the following facts suggesting that the central object is compact and very massive (see e.g. Ref. [1] for a recent review).

(1). Infrared observations of stars moving around the central object in (almost) Keplerian elliptic orbits indicate [2, 3] that the central mass is $M = (4.1 \pm 0.4) \times 10^6 M_\odot$, where M_\odot is the solar mass, and that this mass is located within the pericenter distance of the S2 star, ≈ 0.58 mpc. Expressed in terms of the Schwarzschild radius, R_S , of a black hole with the mass M , this distance is $\sim 1500 R_S$. The mass of matter between the pericenter and apocenter ($\approx 23000 R_S$) distances of S2 does not exceed $\sim 0.1 M$.

(2). Radio observations reveal a strong point-like source, Sgr A*, whose position coincides with the focus of the stellar orbits within the experimental precision, dominated by a systematic error between the infrared and radio coordinate reference frames ($\sim 200 R_S$). Sub-millimeter observations indicate [4, 5] that the angular size of the emitting region is $\sim (30 - 40) \mu\text{as}$, while the expected apparent horizon size of a black hole of mass M is $\approx 52 \mu\text{as}$, taking into account light deflection in the black-hole gravitational field.

(3). While the stars surrounding this radio source move around it at well-detected velocities, the apparent motion of Sgr A* itself may be fully accounted for by the rotation of the Solar System in the Galaxy; in particular, the source does not move, within the measurement precision, perpendicularly to the Galactic plane [6]. This implies [7] that the mass of the body emitting in radio is $M_\star \gtrsim 0.1 M$.

(4). The steady infrared luminosity of Sgr A* is very modest. Compared to much stronger radio emission, which presumably originates from the gas falling to SMCO, this is often used [8, 9] as an argument that the central object cannot have a surface, because the latter would be heated by the falling gas and shine in the infrared ~ 250 times brighter.

(5). The bolometric luminosity of the SMCO is very low, $\sim 3 \times 10^{-9} L_{\text{Edd}}$, compared to the Eddington luminosity L_{Edd} , typical for powerful active galactic nuclei and set by the balance between the accretion flow and the radiation pressure. Partly, this may be attributed to the lack of material for accretion: recent *Chandra* observations indicate [10] that $\sim 10^{-5} M_{\odot}$ of hot gas per year is available for accretion in the SMCO sphere of influence. This amount of matter is insufficient to establish a large-scale accretion disk and would correspond to the Bondi accretion rate of $\sim 10^{-4}$ times the Eddington one. However, the same observations reveal that less than 1% of this captured matter falls to the SMCO, so that the inflow of matter is almost balanced by an outflow.

The extreme compactness of the SMCO, facts (1) and (3) above, as well as the absence of a surface interacting with matter being accreted, fact (4), suggest that the Galactic central object may be a supermassive black hole (SMBH), though it is presently unclear how these objects were initially formed in galactic centers (see e.g. reviews [11, 12] and below). The fact (2), that the radio source looks smaller than horizon, shocking at first sight because one would expect the radio emission to come from an extended accretion disk, may be understood (see e.g. Ref. [13] and references therein) in a model where the source is a small region at the base of a jet. However, the jet model has serious tension [14] with polarization measurements [15]; it may also have tension with the fact (3). More observational data are required to understand definitely the origin of the radio emission.

The fact (5), that is low efficiency of accretion and radiation, may find its explanation in a variety of complicated models of radiatively inefficient accretion flows, see Ref. [16] for a review. Some of them accommodate inflows and outflows similar to those suggested by the *Chandra* results; determination of the actual mechanism of accretion awaits further observations.

All these observational results, while being consistent with the SMBH hypothesis, do not exclude a less compact object without a surface heated by falling baryonic matter. Alternatives to SMBHs have been considered and it has been acknowledged that a compact object made of a scalar field, often called a boson star [17], is perhaps the only acceptable known candidate, see e.g. Ref. [18]. Gravitating Q-balls discussed here represent a subclass of boson stars, though their properties differ drastically from those of classical boson-star solutions kept stable by gravity. The alternatives to SMBHs are often disregarded because of the lack of observational reasons favouring

them against “familiar” black holes, as well as the lack of answers to questions of how they could be formed and why they do not collapse to black holes. All these reasons are not applicable, however, to the Q-ball model discussed below.

Q-balls as SMCOs. — Nontopological solitons, or Q-balls, are compact configurations in scalar field theories whose stability is due to the conservation of a global charge [19–21] (see e.g. a review [22] and textbooks [23, 24]). They received much attention from researchers in field theory and cosmology, but they also have been studied in condensed-matter systems [25]. For scalar potentials satisfying certain conditions, the minimal-energy configuration at a fixed charge Q is compact, that is a Q-ball of charge Q always has lower energy than Q free particles. This is guaranteed by typical power-law dependence of the Q-ball mass M on its charge, $M \propto vQ^A$, where v is some model-dependent dimensionful parameter of the potential and $A \leq 1$ (I use the same notation, M , for the mass of the Galactic SMCO and for a generic Q-ball mass; universal units, $\hbar = c = 1$, are used). The radius of the soliton R is also determined by its charge, $R \propto v^{-1}Q^B$.

These nontopological solitons may be macroscopic classical objects (see, however, an example of particle-like Q-balls in Ref. [26]) and may play a role of SMCOs. If a giant Q-ball resides in the Milky-Way center, it has to satisfy several constraints in order to explain observational data. First, it should not be too compact (and collapse into a black hole), but at the same time should not be too large (and disturb the S2-star results, fact (1)). Only in a few cases explicit solutions for gravitating Q-balls were constructed. However, general arguments suggest [27, 28] (see also Ref. [29]) that the Q-ball collapses to a black hole when its radius R is of order the gravitational radius, $R_S = 2GM$, for its mass M (here $G = 1/M_{\text{Pl}}^2$ is the gravitational constant). We therefore obtain a very general constraint on the parameters of a putative Q-ball in the Milky-Way center, $R_S \leq R \leq 1500R_S$, where the last inequality involves the pericenter distance of the S2 star. Since the $R(Q)$ and $M(Q)$ dependencies relate M and R , this condition may be formulated in terms of the known SMCO mass M and the potential parameter v .

Next, the fact (4) requires that the Q-ball does not have a surface interacting with baryons and emitting heat. This does not mean it should have a horizon: the field ϕ can simply not interact with baryons, or have a very weak interaction. This would make it possible for the accreting matter to penetrate inside (and to go through) the SMCO, thus explaining the absence of thermal emission from the surface. Suppose that the SMCO is much larger than a black hole of the same mass (large compared to the expected size of the accretion disk, say, $R \gtrsim 100R_S$). Most part of the falling baryonic matter simply passes through the Q-ball, providing for an out-

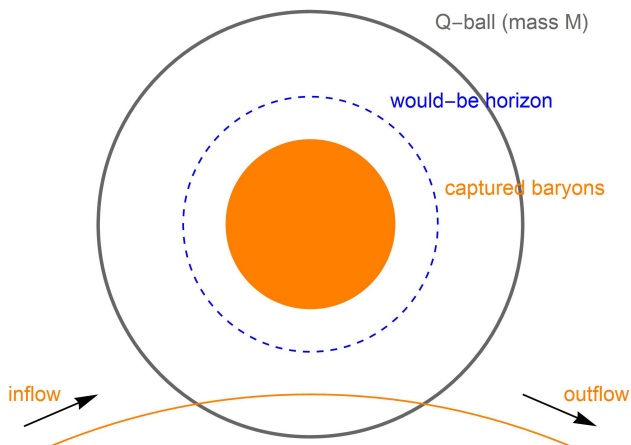


FIG. 1. A sketch of the proposed SMCO. The dashed line represents the horizon which a black hole of mass M would have. The actual SMCO of mass M , the Q-ball, is larger. Baryonic matter may pass through it, giving rise to inflows and outflows, while a small fraction of baryons lose their angular momentum and are captured inside, forming a relatively small radio-emitting blob.

flow which balances the inflow, fact (5). A small fraction of baryons, however, may experience collisions between themselves and lose angular momentum. It is this matter which concentrates in the inner part of the Q-ball and forms the radio source, whose size may be smaller than the horizon size of the would-be black hole with the full SMCO mass, fact (2) (see Fig.1). We note that the possibility to have a radio source of Sgr A* inside a Q-ball was recognized, without a discussion, in Ref. [30], while orbits of test particles penetrating a Q-ball were studied in Ref. [31]. During the lifetime of the Galaxy, the mass of this central baryonic blob grows and may become comparable to the Q-ball mass, fact (3). In fact, the lower limit on the mass of the radio source, fact (3), may be relaxed in this case due to its central location. Eventually, the mass of the entire central object (the Q-ball plus captured baryons) exceeds the stability limit, and the SMCO collapses into SMBH. One may speculate that the drastic change in the accretion process, associated with this transition, triggers the activity of the galaxy, which returns to a quiescent state after stationary accretion to a newborn black hole settles down. This would relate the variety in activity of galactic nuclei to the variety of their central objects and explain why active galaxies are rare, though a detailed study of this prospect is beyond the scope of this Letter.

We see that the Q-balls not only fit all observational constraints on the Galactic SMCO, but may explain easily some of peculiar phenomena seen in the Milky-Way center. I turn now to the question of how these objects might be created in galactic centers. This would relate them to the dark matter.

Birth of supermassive Q-balls. — Recent observations suggest that SMCOs were present in galaxies quite early, with $\sim 10^9 M_\odot$ objects observed at $z \approx 7.1$ [32], $\sim 10^{10} M_\odot$ at $z \approx 6.3$ [33] etc. The standard picture of black-hole formation and growth, implying $\sim 10^2 M_\odot$ seed black holes from stellar explosions, accreting matter and possibly merging, faces difficulties in explaining these observations. It has been suggested that, if dark-matter particles are self-interacting, it helps to solve this problem, either by enhanced accretion rate on the seed black hole [34] or by formation of the supermassive seed itself through the gravothermal collapse [35]. The first mechanism, though efficient, becomes suppressed shortly after it starts to operate, when dark-matter particles start to interact before falling to SMCO [36]. A recent quantitative study [37], based on N -body simulations, suggests that the second mechanism may be viable if a small fraction ($f \lesssim 0.1$ in terms of the density ρ) is “ultra-strongly” interacting dark matter (uSIDM). At a time scale $\sim 10^6$ yr, a central part of the dark-matter halo of mass M_{halo} collapses to a dense object of mass $\sim 0.025 f M_{\text{halo}}$, and the mechanism explains observational data (and the correlation between the SMCO mass and the total halo mass, e.g. Ref. [38]) for $f \lesssim 0.1$ and uSIDM with cross section per unit mass $\bar{\sigma} \equiv \sigma/M \gtrsim (0.3/f) \text{ cm}^2/\text{g}$. The remaining $(1-f)$ of dark matter must have weaker interactions. The value of $\bar{\sigma}$ was constrained, for $f = 1$, from observations of the Bullet Cluster, $\bar{\sigma} \lesssim 0.7 \text{ cm}^2/\text{g}$ [39], and from the ensemble of interacting clusters, $\bar{\sigma} \lesssim 0.47 \text{ cm}^2/\text{g}$ [40], while data on the A320 merging system suggest $\bar{\sigma} \simeq (0.94 \pm 0.06) \text{ cm}^2/\text{g}$ [41]. There are no constraints on $\bar{\sigma}$ for $f \lesssim 0.1$.

Q-balls may work as interacting dark matter [42–44], but application of results of Ref. [37] to them is not straightforward. The interaction cross sections are different for Q-balls of different charges: up to a model-dependent factor of order one, they are geometrical [45, 46], $\sigma \sim \pi R^2$. Since R depends on Q , the population of Q-balls of various charges is not exactly the system studied in Ref. [37]. Next, unlike supposed in Ref. [37], the Q-ball cross section is not purely elastic. Refs. [45, 46] suggest that the elastic and inelastic cross sections are roughly equal, and processes of merging and charge exchange are possible at approximately the same rate as scattering: it is these processes which are responsible, in the end, for the SMCO formation. One might expect that account of these processes, relevant only in the dense central core of a dark-matter halo, would make it even easier to explain the SMCO formation and to solve the cusp-core problem [47] simultaneously; however, only dedicated numerical simulations may give a quantitative description of the corresponding processes. In what follows, we apply the results of Ref. [37] to the Q-ball system, keeping in mind that this would give order-of-magnitude estimates only.

A variety of mechanisms for dark-matter Q-ball pro-

duction have been proposed and studied [42, 48–53]. In all of them, initial charge asymmetry either in the scalar-field condensate or in the ensemble of scalar particles is required, so that there exists a net charge density, subsequently collected and trapped in Q-balls. Therefore, each newborn Q-ball collects its charge from some volume V . The charge asymmetry is defined as $\eta_Q = n_Q/s$, where n_Q is the initial charge density and s is the entropy density at the moment of Q-ball formation. Suppose that the part ξ of the charge is collected into Q-balls of some typical charge Q . Then the number density of Q-balls with the typical charge is $n \sim \xi \eta_Q s/Q$. This may be related to the present-day mass density of Q-balls $\rho_0 = M(Q)n_0 \sim M(Q)\xi \eta_Q s_0/Q$, where $s_0 \sim 3 \times 10^3 \text{ cm}^{-3}$ is the present-day entropy density. On the other hand, the cross section per unit mass is $\bar{\sigma} = \sigma/M(Q) \simeq \pi R(Q)^2/M(Q)$. Eliminating Q , one obtains a relation between ρ_0 and $\bar{\sigma}$, the two key dark-matter parameters.

A particular model. — Consider explicitly a model where Q-balls are produced in the first-order phase transition [24, 50] (see [54] for discussion of other models). It has two scalar fields, a complex one ϕ which Q-balls are made of and a real one χ whose vacuum expectation value gives mass to ϕ . A proper choice of the potential results in a first-order phase transition from the false-vacuum value of χ (massless ϕ) to the true vacuum (heavy ϕ). The ϕ particles are trapped in contracting bubbles of the false vacuum and produce Q-balls.

In this model, the mass scale v is set by the potential difference between the two vacua, $U = v^4$, while the ϕ mass in the true vacuum is $m_\phi = \kappa v$ (in notations of Ref. [50], $U = \lambda v^4$ and $m_\phi = hv$, so $\kappa = h/\lambda^{1/4}$). One has $M = c_M v Q^{3/4}$ and $R = c_R v^{-1} Q^{1/4}$, where $c_M = 4\pi\sqrt{2}/3$ and $c_R = 1/\sqrt{2}$. The Sgr A* constraint discussed above requires $1.3 \text{ keV} \lesssim v \lesssim 180 \text{ keV}$, with values $v \lesssim 6 \text{ keV}$ favored for explanation of the weak accretion.

For the Q-balls to play the role of uSIDM, there are two conditions: $f \equiv \rho_0/\rho_{\text{DM}} \lesssim 0.1$, where $\rho_{\text{DM}} \sim 10^{-6} \text{ GeV/cm}^3$ is the present-day dark-matter density, and $\bar{\sigma} f \gtrsim 0.3 \text{ cm}^2/\text{g}$. The two constraints may be satisfied simultaneously for a certain choice of parameters (ξ, η_Q) provided $v(\xi \eta_Q)^{1/4} \lesssim 100 \text{ keV}$, which agrees well with the Sgr A* constraints.

This model allows also for two interesting possibilities to include the remaining $(1 - f)$ fraction of the dark matter. First, the remaining $(1 - \xi)$ excess of charge, not trapped into Q-balls, is kept in the form of ϕ particles. They are stable because they are the lightest particles charged under the global $U(1)$ symmetry responsible for the Q-ball stability. Depending on their mass and interactions, they may represent the dominant part of the present dark-matter density.

Second, Q-balls of different sizes are produced, and, since the cross section of a Q-ball depends on its charge, their cross sections also vary. The bulk of produced Q-balls may form the standard dark matter, while a small

fraction of them (more precisely, those Q-balls which contribute a small fraction to ρ) play the role of uSIDM. It is possible to estimate [54] the distribution of the Q-balls in Q ; while small Q-balls are born more frequently, they carry a minor fraction of ρ . Requiring $\rho_0 \sim \rho_{\text{DM}}$ and $\bar{\sigma} \lesssim 1 \text{ cm}^2/\text{g}$, one obtains the bound $v \gtrsim 240 \text{ keV}$. Recalling that the cosmological constraint was a rough order-of-magnitude estimate only, we conclude that the model with $v \sim 100 \text{ keV}$ may be capable of producing the dark-matter Q-balls and the Q-ball Milky-Way SMCO at the same time. However, it remains to be studied how the Q-ball of radius $R \lesssim 10R_S$ would change the accretion process with respect to a black hole.

Clearly, detailed quantitative studies, which might require full-scale computer simulations, are necessary to fully understand details of birth and subsequent evolution of the dark-matter Q-ball system. However, our order-of-magnitude estimates demonstrate that this interesting scenario may be viable.

Future tests of the model. — It would be difficult to test the proposed scenario by direct searches for the dark-matter Q-balls, e.g. [55]: their number density in the Universe is very low while the interaction with normal matter is very weak, if any. Detailed studies in particular models might reveal observable signatures of intermediate-mass Q-balls, if they are produced. Model-dependent signatures may be found also for gravitational-wave astronomy, coming both from the primordial formation of Q-balls [56] and from the SMCO [31]. However, definitive tests will be provided by high-resolution observations of the Milky-Way central object.

For instance, when the G2 dusty object [57] has been discovered on its way to the Milky-Way SMCO, orders-of-magnitude increase in the accretion rate, and a corresponding burst of luminosity of Sgr A*, were predicted. This extended object has been already observed after passing the pericenter, tidally disturbed but without any sign of increased accretion [58–61]. Further observations will help to understand its nature and to shed light on the details of the accretion process.

The Event Horizon Telescope [62], an Earth-size radio interferometer working at the frequency of 1.3 mm, might be able to resolve the black-hole shadow, if the Milky-Way SMCO is a black hole, in a few years. Future “Millimetron” spaceborn interferometer [63] would be able to have horizon-scale resolution for dozens of nearby SMCOs, and its observations would establish the nature of supermassive objects in galactic centers.

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Supplementary Information

Distribution of Q-balls in Q after the first-order phase transition. —

Let us estimate the distribution of Q-balls in Q to see that, while small Q-balls are born more frequently, they carry a minor fraction of ρ . The charge of a newborn Q-ball is determined by the volume of the remaining bubble of the old phase. The size of suitable bubbles may be estimated from the condition that a smaller bubble of the new phase is not created inside before the bubble collapses. The probability that a new-phase bubble is created in the volume of the old-phase bubble V is proportional to V times the time of the bubble collapse, that is to $V \times V^{1/3} = V^{4/3} \propto Q^{4/3}$. Therefore, the probability to create a Q-ball with charge Q , and hence the number density of these Q-balls (the number of Q-balls with the charge Q per unit volume),

$$n(Q) \propto 1 - \left(\frac{Q}{Q_*} \right)^{4/3},$$

where Q_* is some characteristic (maximal) charge estimated in Ref. [50]. This expression works for $Q \gtrsim Q_{\min}$, where $Q_{\min} \ll Q_*$ is the minimal charge of a stable Q-ball. This means that most of the Q-balls are born with small charges, $Q \sim Q_{\min}$. However, their contribution to the mass density is small,

$$\rho(Q) \propto M(Q)n(Q) \propto Q^{3/4} \left(1 - (Q/Q_*)^{4/3} \right).$$

This function has a maximum at $Q \sim 0.5Q_* \gg Q_{\min}$, and most of the density is carried by these large Q-balls with relatively small $\bar{\sigma}$.

Let us divide artificially the Q-balls born after the phase transition into two populations; those with $Q_{\min} \leq Q \leq Q_0$, for some Q_0 , would represent uSIDM, while those with $Q_0 \leq Q \leq Q_*$ would be the bulk dark matter. Then, we should simultaneously satisfy the following constraints:

- the value of $\bar{\sigma}f$, summed over uSIDM, $\langle \bar{\sigma}f \rangle_{\text{u}} \gtrsim 0.3 \text{ cm}^2/\text{g}$;
- the fraction of uSIDM $\langle f \rangle_{\text{u}} \lesssim 0.1$;
- the mean cross section of the bulk dark matter $\langle \bar{\sigma} \rangle_{\text{b}} \lesssim 1 \text{ cm}^2/\text{g}$.

Replacing sums over Q by integrals, one finds

$$\langle \bar{\sigma}f \rangle_{\text{u}} = \bar{\sigma}_* \frac{\int_{x_{\min}}^{x_0} x^{-1/4} x^{3/4} (1 - x^{4/3}) dx}{\int_{x_{\min}}^1 x^{3/4} (1 - x^{4/3}) dx},$$

where $x = Q/Q_*$, $x_0 = Q_0/Q_*$, $x_{\min} = Q_{\min}/Q_*$ and $\bar{\sigma}_* = \bar{\sigma}(Q_*)$. For $x_{\min} \ll x_0 \ll 1$, one has $\langle \bar{\sigma}f \rangle_{\text{u}} \approx 2.7x_0^{3/2}\bar{\sigma}_*$. Similarly,

$$\langle f \rangle_{\text{u}} = \frac{\int_{x_{\min}}^{x_0} x^{3/4} (1 - x^{4/3}) dx}{\int_{x_{\min}}^1 x^{3/4} (1 - x^{4/3}) dx} \approx 2.3x_0^{7/4}$$

and

$$\langle \bar{\sigma} \rangle_{\text{b}} = \bar{\sigma}_* \frac{\int_{x_0}^1 x^{-1/4} x^{3/4} (1 - x^{4/3}) dx}{\int_{x_{\min}}^1 x^{3/4} (1 - x^{4/3}) dx} \approx (1.3 - 2.7x_0^{3/2})\bar{\sigma}_*.$$

For $x_0 \lesssim 0.17$ and $\bar{\sigma}_* \lesssim 2.6 \text{ cm}^2/\text{g}$, all three conditions are satisfied. As is shown in the main text, for $v \sim 100 \text{ keV}$, correct values of $\bar{\sigma}$, which we required here, correspond to the correct density of dark matter while Sgr A* constraints are also satisfied.

General models. —

Numerous potentials allowing for Q-balls have been suggested. To the best of author's knowledge, all of the studied models lead to particular values of the power-law exponents A, B in $M \propto Q^A$, $R \propto Q^B$ relations, namely, $A = (6 - \alpha)/(2(4 - \alpha))$ and $B = (2 - \alpha)/(2(4 - \alpha))$ for a certain α , $-2 \leq \alpha \leq 2$.

The models with $0 \leq \alpha \leq 2$ are relevant for the case of “almost flat” potentials which behave as $|\phi|^\alpha$ at large $|\phi|$, see e.g. [42]. It is easy to obtain explicit expressions for the general case.

Mass:

$$M(Q) = c_M v Q^{\frac{6-\alpha}{2(4-\alpha)}},$$

where

$$c_M = 2\pi \frac{4 - \alpha}{3 - \alpha} \left(\frac{(3 - \alpha)}{12} \right)^{\frac{1}{4-\alpha}}.$$

Radius:

$$R(Q) = c_R v^{-1} Q^{\frac{2-\alpha}{2(4-\alpha)}},$$

where

$$c_R = \frac{1}{2} \left(\frac{(3 - \alpha)}{12} \right)^{-\frac{1}{4-\alpha}}.$$

The Sgr A* constraint:

$$\frac{F}{1500} \lesssim \frac{v}{M_{\text{Pl}}} \lesssim F,$$

where $M_{\text{Pl}} \approx 1.2 \times 10^{19} \text{ GeV}$ is the Planck mass and

$$F = c_M^{-\frac{2-\alpha}{2(4-\alpha)}} c_R^{\frac{6-\alpha}{2(4-\alpha)}} \left(\frac{M}{M_{\text{Pl}}} \right)^{-\frac{2}{4-\alpha}}.$$

Cross section per unit mass:

$$\bar{\sigma} = c_\sigma v^{-3} Q^{-\frac{2+\alpha}{2(4-\alpha)}},$$

where

$$c_\sigma = \frac{3 - \alpha}{4 - \alpha} \left(\frac{(3 - \alpha)}{12} \right)^{-\frac{3}{4-\alpha}}$$

(note that $\bar{\sigma}$ always decreases with Q).

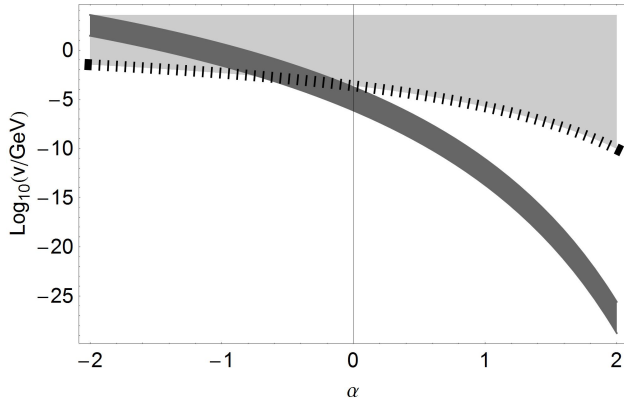


FIG. 2. Constraints on the scale parameter v of the scalar potential for the case when all dark matter is made of Q-balls. The dark gray band represents the values required for a Q-ball SMCO in the Milky Way. The light-gray shaded area is allowed by constraints derived from the dark-matter density and cross section. The latter constraints are order-of-magnitude estimates. A $f \lesssim 0.1$ fraction of Q-balls interacting considerably stronger and responsible for SMCO formation is allowed.

Density – cross section relation:

$$\rho_0 = c_M v \left(\frac{\bar{\sigma} v^3}{c_\sigma} \right)^{\frac{2-\alpha}{2+\alpha}} \eta_Q s_0.$$

Note that the $\alpha = 0$ case also corresponds to a completely different model of Refs. [20, 50] discussed in the main text. The values of c_M , c_R and c_σ for the model of Ref. [28] with $\alpha = -2$ are slightly different, $c_M = 3\pi$, $c_R = 1/2$, $c_\sigma = 1/12$, but the difference in numerical constraints cannot be seen by eye in further plots.

Consider two scenarios.

(i). The bulk of produced Q-balls forms the standard dark matter, while a small fraction of them (more precisely, those Q-balls which contribute a small fraction to ρ) play the role of uSIDM. Then, one should require $\rho_0 \sim \rho_{\text{DM}} \sim 10^{-6} \text{ GeV/cm}^3$ and $\bar{\sigma} \lesssim 1 \text{ cm}^2/\text{g}$. Since $\eta_Q < 1$ by definition, we obtain a *lower* limit on v .

(ii). The bulk of Q-balls forms uSIDM while most of the dark matter is represented by a completely different component. This requires $\rho_0 = f \rho_{\text{DM}}$ with $f \lesssim 0.1$ and $\bar{\sigma} f \gtrsim 0.3 \text{ cm}^2/\text{g}$. The resulting *upper* limits on v depend now on the assumed η_Q .

Figures 2 and 3 illustrate constraints on scenarios (i) and (ii), respectively. The dark grey band in both plots represents the values required by the Sgr A* constraints. In Fig. 2, the area of parameters allowed for scenario (i) is shown as a light grey region. Constraints on scenario (ii) cannot be formulated unless a particular value of the charge asymmetry η_Q is assumed; they put upper limits on v shown in Fig. 3 for various values of η_Q .

Comments on particular models. — (A). Flat scalar potential, $U(\phi) \sim \text{const}$ at large $|\phi|$, $\alpha = 0$ [21, 42].

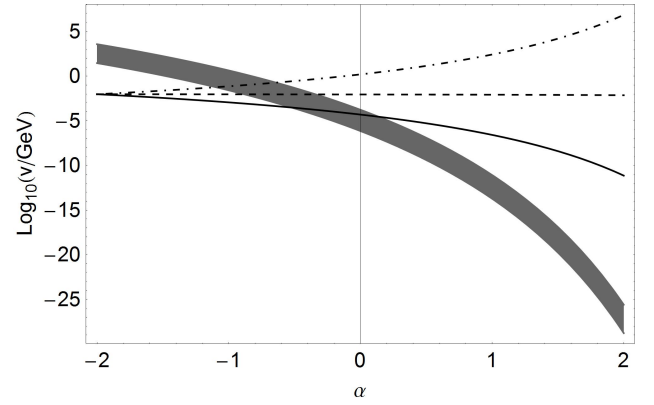


FIG. 3. Constraints on the scale parameter v of the scalar potential for the case when Q-balls represent only the fraction of dark matter responsible for SMCO formation by gravothermal collapse. The dark gray band represents the values required for a Q-ball SMCO in the Milky Way. Thin lines represent *upper* limits on v for the charge asymmetry $\eta_Q = 1$ (solid), 10^{-9} (dashed) and 10^{-18} (dash-dotted). The latter constraints are order-of-magnitude estimates.

Q-balls form from the ϕ condensate which develops instabilities. This process is highly nonlinear, and numerical simulations are required to study it. It has been shown [51] that most part of the charge is collected to large Q-balls (low $\bar{\sigma}$), while a number of small Q-balls (large $\bar{\sigma}$) are also produced. This mechanism may, in principle, work in scenario (i), though quantitative results on the distribution of produced Q-balls in Q are presently missing.

(B). Second-order phase transition, $\alpha = 0$ [48, 49]. The probability to create a Q-ball is determined by fluctuations and is exponentially small. As a result, only a small fraction of charge is collected to Q-balls with $Q \sim Q_{\text{min}}$, with exponentially suppressed chance to create a larger Q-ball. This model may work only in scenario (ii) because almost all Q-balls have the same size and hence the same cross section.

(C). Logarithmic potential, $\alpha = 2$ [52, 53]. The production of these Q-balls was studied numerically in more detail and the distribution of Q-balls in Q was presented in Ref. [53]. However, in this model, $R = \text{const}$ and does not depend on Q , while $M \propto Q$. As a consequence, only a very light scalar field may prevent collapsing of a massive Q-ball to a black hole, cf. Figs. 2, 3 (scenario (ii) is allowed for $m \sim 10^{-26} \text{ GeV}$ and scenario (i) is excluded).

(D). Model of Ref. [28], $\alpha = -2$. Here, $M \propto Q^{2/3}$ and $R \propto Q^{1/3}$, so $\bar{\sigma}$ does not depend on Q . This might work for scenario (ii) only, but is excluded by Fig. 3. Note also that no mechanism to produce Q-balls in the early Universe is known for this model, best studied in the context of boson stars.